NS8-1 Factors and Multiples

The **multiples** of a number are the numbers you say when counting by that number.

$3 \times 5 = 15$ — 15 is a **multiple** of both 3 and 5

3 and 5 are both **factors** of 15

$0 \times 4 = 0$ — 0 is a **multiple** of both 0 and 4

0 and 4 are both **factors** of 0

1. List the first few multiples of these numbers.

a) 3: *0*, *3*, *6*, ____, ____, ____, ____

b) 4: ____, ____, ____, ____, ____, ____, ____

c) 5: ____, ____, ____, ____, ____, ____, ____

2. Look at the lists you made in Question 1.

a) Is 12 a multiple of 4? _____ How do you know? _____

b) Is 17 a multiple of 5? _____ How do you know? _____

c) Is 0 a multiple of 3? _____ Of 4? _____ Of 5? _____

3. a) Write 0 as a multiple of 17: $0 = 17 \times$ _____

b) Which whole numbers is 0 a multiple of? Explain. _____

_____

4. Rewrite each statement in a way that means the same thing but uses the word "factor."

a) 20 is a multiple of 5. _____

b) 9 is a multiple of 1. _____

c) 0 is a multiple of 8. _____

d) 8 is a multiple of 8. _____

e) 11 is not a multiple of 4. _____

f) Every number is a multiple of 1. _____

g) Every number is a multiple of itself. _____

h) 0 is a multiple of any number. _____

5. Rewrite each statement in a way that means the same thing but uses the word "multiple."

a) 5 is a factor of 15.

b) 2 is a factor of 18.

c) 3 is a factor of 0.

d) Every number is a factor of 0.

e) 1 is a factor of 7.

f) 1 is a factor of every number.

g) 6 is a factor of 6.

h) Any number is a factor of itself.

6. Alana wants to find all pairs of numbers that multiply to give 10.

She lists each number from 1 to 10 in a chart. She looks for a second number that multiplies with the first to give 10.

a) Why didn't Alana list any number greater than 10 in the first column of her table?

b) Why didn't Alana list 0 in the first column of her table?

1st	2nd
1	10
2	5
3	
4	
5	2
6	
7	
8	
9	
10	1

7. Use Alana's method to find all pairs of numbers that multiply to give the number in bold.

a) **6**

1st	2nd
1	
2	
3	
4	
5	
6	

b) **8**

1st	2nd
1	
2	
3	
4	
5	
6	
7	
8	

c) **9**

1st	2nd
1	
2	
3	
4	
5	
6	
7	
8	
9	

8. Cross out the pairs that are repeated in Question 7.

9. Connor makes a chart to list all the factors of 20. He doesn't want to write and check all the numbers from 1 to 20. He starts his list as follows:

a) Connor knows that $5 \times 4 = 20$. He thinks that if $6 \times \square = 20$, then $\square$ must be less than 4. Explain his thinking.

b) Explain why Connor's list is complete.

1st	2nd
1	20
2	10
3	
4	5
5	4

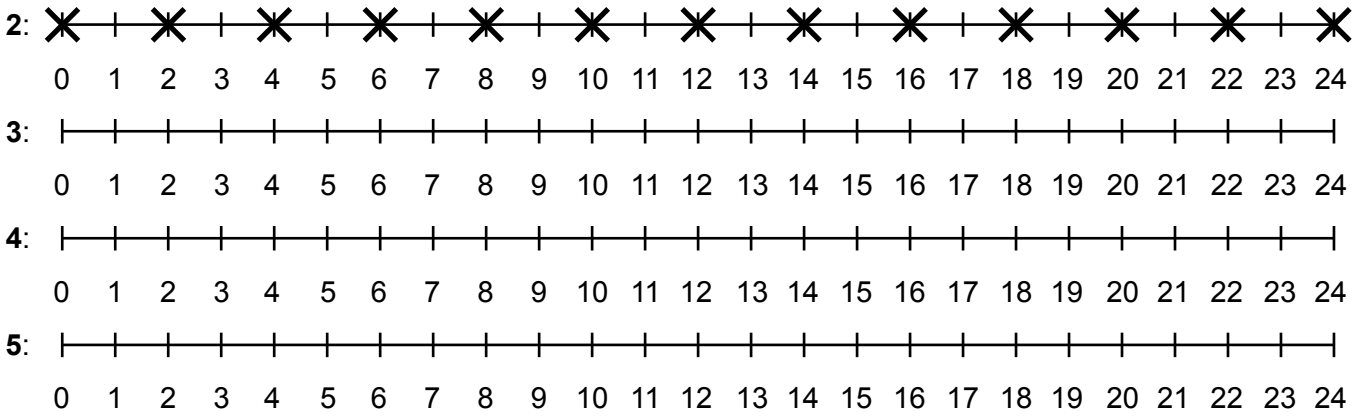
10. Connor used this chart to help him identify pairs that multiply to 36. Why did he know that his search was complete as soon as he found a pair with both numbers the same?

1st	2nd
1	36
2	18
3	12
4	9
5	
6	6

11. Find all pairs of numbers that multiply to 120.

NS8-2 LCMs and GCFs

1. Mark the multiples of each number on the number lines.



2. 0 is a multiple of every number. Not counting 0, find the first 2 common multiples of:

a) 2 and 5 ____, ____ b) 2 and 3 ____, ____ c) 3 and 4 ____, ____ d) 2 and 4 ____, ____

The **lowest common multiple (LCM)** of two or more numbers is the smallest number (not 0) that is a multiple of the numbers.

3. Look at your answers to Question 2. What is the LCM of:

a) 2 and 5 ____ b) 2 and 3 ____ c) 3 and 4 ____ d) 2 and 4 ____

4. Find the lowest common multiple of each pair of numbers.

a) 3 and 5
3: *3*, *6*, *9*, *12*, ***15***, *18*
5: *5*, *10*, ***15***, *20*
LCM = ***15***

b) 6 and 10
6:
10:
LCM = ____

c) 9 and 12
9:
12:
LCM = ____

d) 2 and 6
2:
6:
LCM = ____

e) 2 and 10 f) 2 and 9 g) 3 and 15 h) 4 and 8 i) 8 and 8

j) 5 and 15 k) 5 and 10 l) 3 and 10 m) 6 and 15 n) 6 and 8

5. a) How can you find the second common multiple of two numbers from the first?

b) The first common multiple of 18 and 42 is 126. What is the second common multiple? _____

6. Find all the factors of each number by dividing the number by the whole numbers in increasing order—divide by 1, 2, 3, 4, 5, and so on. How do you know when you can stop dividing?

a) 33
1, 3, 11, 33

b) 55 c) 65 d) 66 e) 90

The greatest number that is a factor of two or more numbers is called the **greatest common factor (GCF)** of the numbers.

7. Use your answers to Question 6. Find the greatest common factor of:

a) 33 and 55 b) 33 and 66 c) 33 and 90 d) 65 and 66

e) 33 and 65 f) 55 and 65 g) 33, 55 and 65 h) 55, 65 and 90

Two numbers are called **consecutive** if one number is the next number after the other.

Example: 13 and 14 are consecutive because 14 is the next number after 13.

INVESTIGATION 1 ▶ What is the GCF of two consecutive numbers?

A. Find the factors of each number and then the GCF of each pair.

a) 14 and 15
14: 1, 2, 7, 14
15: 1, 3, 5, 15
GCF: *1*

b) 24 and 25
24:
25:
GCF: ____

c) 27 and 28
27:
28:
GCF: ____

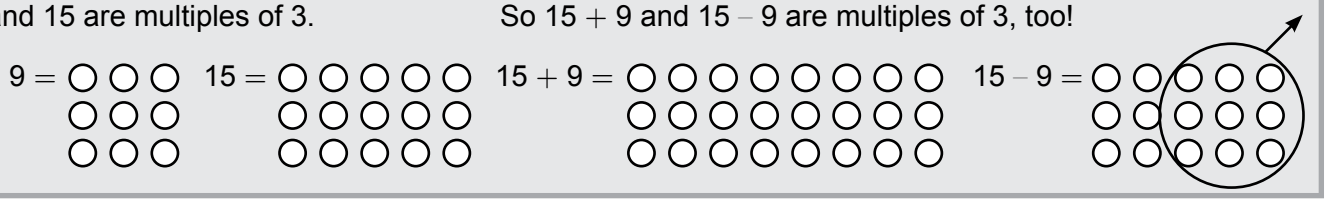
d) 44 and 45
44:
45:
GCF: ____

B. Make a conjecture about the GCF of any two consecutive numbers.

_____

C. Test your conjecture on two consecutive numbers of your choice: ____ and ____ GCF: ____

9 and 15 are multiples of 3. So $15 + 9$ and $15 - 9$ are multiples of 3, too!



8. a) Rewrite the conclusion in the box using the word factor instead of multiple:

3 is a factor of both 9 and 15, so 3 is a factor of both _____ *and* _____

b) Draw pictures to show that any factor of both 8 and 20 is also a factor of both $20 + 8$ and $20 - 8$.

c) Explain why any common factor of 99 and 100 must divide the sum $99 + 100$.

d) Explain why any common factor of 99 and 100 must divide the difference $100 - 99$.

e) Without finding the factors of 99 and 100, explain why their GCF is 1.

INVESTIGATION 2 ▶ How are the GCF, the LCM, and the product of two numbers related?

A. Complete the chart. Include three more values of your choice for *a* and *b*.

a	***b***	***a* × *b***	**GCF**	**LCM**	**GCF × LCM**
3	4				
2	5				
4	6				
10	15				
5	10				
3	5				
4	5				
6	9				
12	15				

B. Which two columns are the same in every row?

_____ and _____

C. Write an expression for the LCM in terms of *a* × *b* and GCF.

LCM = _____

D. When the LCM is the same as the product, what is the GCF? ____

E. Choose two more pairs of numbers *a* and *b* where *a* is a factor of *b*, and complete the chart.

a	***b***	***a* × *b***	**GCF**	**LCM**	**GCF × LCM**
2	6				

Which columns are equal? GCF = ____, LCM = ____ and GCF × LCM = ____ × ____

NS8-3 Prime Numbers

A **prime** number has **exactly two** distinct factors: itself and 1.

A **composite** number has **more than two** distinct factors: itself, at least one number other than itself, and 1.

1. How many distinct factors does the number 1 have? _____ Is 1 a prime number? _____

2. List all prime numbers less than 10: _____

3. List all composite numbers between 10 and 20: _____

4. What is the greatest prime number less than 30? _____

5. Circle the prime numbers in each list.

a) 5 4 2 8 9 1

b) 6 2 3 4 7 10

c) 11 25 14 13 17 20

d) 27 15 12 18 29 33

6. List all the factors of each number.

a) 25: *1, 5, 25*

b) 8: _____

c) 12: _____

d) 16: _____

e) 9: _____

f) 18: _____

g) 50: _____

h) 45: _____

i) 60: _____

j) 42: _____

7. Put a check mark in front of the numbers that are composite numbers.

_____ 30 _____ 31 _____ 32 _____ 33 _____ 34 _____ 35 _____ 36 _____ 37

8. Write a number between 0 and 20 that has…

a) two factors _____ b) four factors _____ c) five factors _____

9. The prime numbers 3 and 5 differ by 2. Find three other pairs of prime numbers less than 20 that differ by 2:

10. Write three consecutive numbers which are also all composite numbers:

11. Eratosthenes was a Greek scholar who was born over 2 000 years ago in what is now Libya. He developed a method to systematically identify prime numbers. It is called **Eratosthenes' Sieve**.

Follow these directions to use Eratosthenes' Sieve:

a) Shade the number 1 (it is not prime).

b) Circle 2, 3, 5, and 7—all the primes less than 10.

c) Shade all the remaining multiples of 2.

d) Shade all the remaining multiples of 3.

e) Shade all the remaining multiples of 5.

f) Shade all the remaining multiples of 7.

g) Circle the next uncircled number (11).

Note that all multiples of 11 less than 100 (11×2, 11×3, …, 11×9) are **already shaded** because they have a factor less than 10.

h) Circle the next uncircled number. How do you know all multiples of that number less than 100 are already shaded?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

_____

_____

i) Now circle all the remaining numbers.

You've just used Eratosthenes' Sieve to circle all the prime numbers from 1 to 100!

12. How many prime numbers are there between 30 and 50? _____

13. Solve these riddles.

a) I am a prime number less than 100. If you add 10 or 20 to me, the result is prime. What number am I?

b) I am a prime number less than 100. My digits add to 13. What number am I?

c) I am a prime number less than 100. My tens digit is one more than my ones digit. What number am I?

d) I am a prime number between 20 and 70. If you reverse my digits, the result is a larger prime number. What number am I?

NS8-4 Prime Factorizations

Any **composite** number can be written as a product of prime numbers. This product is called the **prime factorization** of the original number.

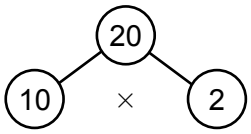
10×2 **is not** a prime factorization of 20 because the number 10 is composite

$5 \times 2 \times 2$ **is** a prime factorization of 20

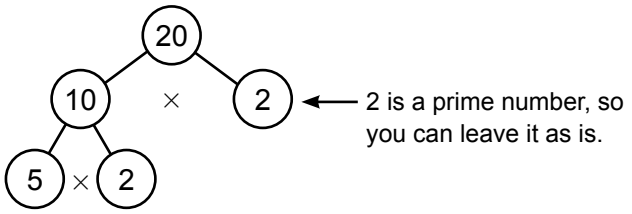
You can find a prime factorization for a number by using a **factor tree**.

Here is how you can make a factor tree for the number 20:

Step 1: Find any pair of numbers (not including 1) that multiply to give 20.

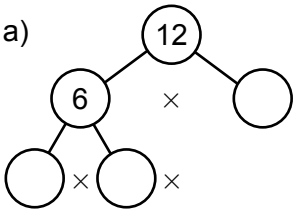


Step 2: Repeat Step 1 for the numbers on the "branches" of the tree.

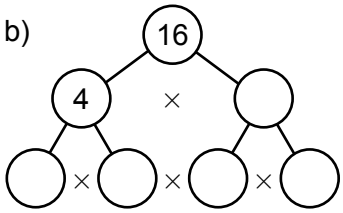


1. Complete the factor trees.

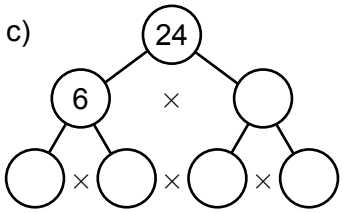
a)



b)



c)



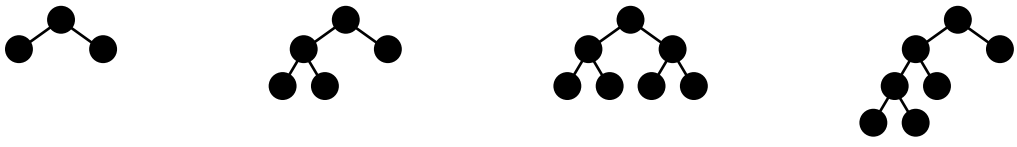
2. Write a prime factorization for each number.

a) $20 = 10 \times 2 = 2 \times 5 \times 2$ b) $18 =$ c) $8 =$ d) $14 =$

3. Use a factor tree to find a prime factorization for each number.

a) 30 b) 36 c) 27 d) 28 e) 75

4. Here are some **branching patterns** for factor trees:



Can you find a factor tree for the number 24 that has a different branching pattern from the tree in Question 1 c)?

NS8-5 Prime Factorizations and GCFs

INVESTIGATION ▶ How does the prime factorization of a number compare to the prime factorization of its factors?

A. Write the prime factorization of 72 and all its factors:

Factors of 72	Prime Factorization
1	____
2	*2*
3	*3*
4	*2 × 2*
6	
8	
9	
12	
18	
24	
36	
72	

B. How many 2s are in the prime factorization of 72? _____

Does any factor of 72 have more 2s in its prime factorization than 72 does? _____

C. How many 3s are in the prime factorization of 72? _____

Does any factor of 72 have more 3s in its prime factorization than 72 does? _____

D. Finish the sentences below by writing **at least** or **at most**.

Any factor of 72 must have _____ as many 2s in its prime factorization as 72 does.

Any factor of 72 must have _____ as many 3s in its prime factorization as 72 does.

E. Does 72 have a 5 in its prime factorization?

Does any factor of 72 have a 5 in its prime factorization? Explain why this is so.

---

1. The prime factorization of 180 is $2 \times 2 \times 3 \times 3 \times 5$. Without doing any calculations, circle the products that show factors of 180:

$2 \times 3 \times 5$ $2 \times 3 \times 7$ $2 \times 3 \times 3 \times 5$ $3 \times 3 \times 3$ 5×5

How did you decide which products to circle?

2. a) Find the prime factorizations of 84 and 96. Do the rough work in your notebook.

$84 =$ ___2___ $\times$ ___2___ $\times$ ___3___ $\times$ ___7___

$96 =$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____

b) Any factor of 84 must have in its prime factorization **at most**:

___*two*___ 2s, ___*one*___ 3s, ___*one*___ 7s

c) Any factor of 96 must have in its prime factorization **at most**:

_____ 2s, _____ 3s

d) Can a common factor of 84 and 96 have any 7s in its prime factorization? _____

How do you know? _____

e) Any common factor of 84 and 96 must have in its prime factorization **at most**:

_____ 2s, _____ 3s

f) The prime factorization of the **greatest common factor (GCF)** of 84 and 96 is:

_____ $\times$ _____ $\times$ _____ So the GCF of 84 and 96 is _____.

3. The prime factorization of each number is given. Match up as many pairs of common prime factors as you can. Then find the prime factorization of the GCF and calculate the GCF.

a) $36 = 2 \times 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

GCF = ___2___ $\times$ ___2___ $\times$ ___3___ $=$ ___12___

b) $60 = 2 \times 2 \times 3 \times 5$

$50 = 2 \times 5 \times 5$

GCF = _____ $\times$ _____ $=$ _____

c) $42 = 2 \times 3 \times 7$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

GCF = _____ $\times$ _____ $=$ _____

d) $90 = 2 \times 3 \times 3 \times 5$

$140 = 2 \times 2 \times 5 \times 7$

GCF = _____ $\times$ _____ $=$ _____

4. Write a prime factorization for each number, then find the GCF of each pair.

a) 24 and 32 b) 24 and 30 c) 16 and 40 d) 27 and 39 e) 70 and 56

5. Find the GCF of the numbers.

a) 24, 30, 54 b) 84, 210, 300 c) 45, 72, 120

NS8-6 Prime Factorizations and LCMs

INVESTIGATION ▶ How does the prime factorization of a number compare to the prime factorization of its multiples?

A. Write the prime factorizations of the first ten multiples of 90 (don't include zero).

Multiples of 90	Prime factorizations
1×90	$2 \times 3 \times 3 \times 5$
2×90	$\mathbf{2} \times 2 \times 3 \times 3 \times 5$
3×90	$\mathbf{3} \times 2 \times 3 \times 3 \times 5$
4×90	$\mathbf{2} \times \mathbf{2} \times 2 \times 3 \times 3 \times 5$
5×90	
6×90	
7×90	
8×90	
9×90	
10×90	

B. How many 2s are in the prime factorization of 90? _____

Does any multiple of 90 have fewer 2s in its prime factorization than 90 does? _____

C. How many 3s are in the prime factorization of 90? _____

Does any multiple of 90 have fewer 3s in its prime factorization than 90 does? _____

D. How many 5s are in the prime factorization of 90? _____

Does any multiple of 90 have fewer 5s in its prime factorization than 90 does? _____

E. Finish the sentences below by writing **at least** or **at most**.

Any multiple of 90 must have _____ as many 2s in its prime factorization as 90 does.

Any multiple of 90 must have _____ as many 3s in its prime factorization as 90 does.

Any multiple of 90 must have _____ as many 5s in its prime factorization as 90 does.

F. Does 90 have a 7 in its prime factorization? _____

Does any multiple of 90 have a 7 in its prime factorization? _____

---

1. The prime factorization of 60 is $2 \times 2 \times 3 \times 5$.

Without doing any calculations, circle the products that show multiples of 60:

$2 \times 2 \times 3 \times 3 \times 5$ $2 \times 3 \times 5 \times 7 \times 7$ $2 \times 2 \times 5 \times 5 \times 5$ $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 11$

How did you decide which products to circle?

2. a) Find the prime factorizations of 90 and 168. Do the rough work in your notebook.

$90 =$ ___2___ $\times$ ___3___ $\times$ ___3___ $\times$ ___5___

$168 =$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____

b) Any multiple of 90 must have in its prime factorization **at least**:

___*one*___ 2s, ___*two*___ 3s, ___*one*___ 5s

c) Any multiple of 168 must have in its prime factorization **at least**:

_____ 2s, _____ 3s, _____ 7s

d) Any common multiple of 90 and 168 must have in its prime factorization **at least**:

_____ 2s, _____ 3s, _____ 5s, and _____ 7s.

e) The **lowest common multiple (LCM)** of 90 and 168 must be:

_____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $=$ _____

3. a) Find the prime factorizations of 100 and 126.

$100 =$ _____ $\times$ _____ $\times$ _____ $\times$ _____

$126 =$ _____ $\times$ _____ $\times$ _____ $\times$ _____

b) Any multiple of 100 must have in its prime factorization **at least**:

_____ 2s, and _____ 5s

c) Any multiple of 126 must have in its prime factorization **at least**:

_____ 2s, _____ 3s, and _____ 7s

d) Any common multiple of 100 and 126 must have in its prime factorization **at least**:

_____ 2s, _____ 3s, _____ 5s, and _____ 7s.

e) The **lowest common multiple (LCM)** of 100 and 126 must be:

_____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $=$ _____

4. The prime factorization of each number is given. Find the prime factorization of the LCM. Then calculate the LCM.

a) $90 = 2 \times 3 \times 3 \times 5$ and $140 = 2 \times 2 \times 5 \times 7$

So LCM $=$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $=$ _____

b) $120 = 2 \times 2 \times 2 \times 3 \times 5$ and $180 = 2 \times 2 \times 3 \times 3 \times 5$

So LCM $=$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $\times$ _____ $=$ _____

5. Find the prime factorizations of each number. Then find the prime factorization of the LCM and calculate the LCM.

a) 35 and 84 b) 15 and 21 c) 50 and 60 d) 42 and 72 e) 24 and 48

NS8-7 Order of Operations

Addition and subtraction are done from left to right. If there are brackets, do the operations in brackets first. Example: $7 - 3 + 2 = 4 + 2 = 6$ but $7 - (3 + 2) = 7 - 5 = 2$

1. a) Calculate each expression using the correct order of operations.

$(12 + 9) - 2 - 1$ $\quad$ $12 + (9 - 2) - 1$ $\quad$ $12 + 9 - (2 - 1)$

$(12 + 9 - 2) - 1$ $\quad$ $12 + (9 - 2 - 1)$ $\quad$ $(12 + 9) - (2 - 1)$

b) How many different answers did you get in part a)? _____

2. a) Add brackets in different ways to get as many different answers as you can.

i) $12 + 9 + 2 + 1$ $\quad$ ii) $12 - 9 + 2 - 1$ $\quad$ iii) $12 - 9 - 2 - 1$

b) How many different answers did you get in part a)? i) _____ ii) _____ iii) _____

c) Check all that apply. The order of operations affects the answer when the expression consists of…

☐ addition only $\quad$ ☐ subtraction only $\quad$ ☐ addition and subtraction

Multiplication and division are done from left to right. If there are brackets, do the operations in brackets first. Example: $15 \div 5 \times 3 = 3 \times 3 = 9$ but $15 \div (5 \times 3) = 15 \div 15 = 1$

3. Evaluate each expression.

a) $4 \times 3 \div 6 \times 7$ $\quad$ b) $6 \times 4 \div 2 \div 3$ $\quad$ c) $30 \div 5 \div (2 \times 3)$ $\quad$ d) $16 \times 2 \div (4 \times 2)$

4. a) Add brackets in different ways to get as many different answers as you can.

i) $2 \times 3 \times 2 \times 5$ $\quad$ ii) $64 \div 8 \div 4 \div 2$ $\quad$ iii) $90 \div 5 \times 6 \div 3$

b) Which expressions in part a) give the same answer, no matter where you place the brackets?

5. Do the operation in brackets first.

a) $18 + (6 \times 3)$ $\quad$ b) $(18 + 6) \times 3$ $\quad$ c) $(18 + 6) \div 3$ $\quad$ d) $18 + (6 \div 3)$

$= 18 + 18 = 36$

e) $18 - (6 \times 3)$ $\quad$ f) $(18 - 6) \times 3$ $\quad$ g) $(18 - 6) \div 3$ $\quad$ h) $18 - (6 \div 3)$

6. Check all that apply. The order of operations affects the answer when the expression combines…

☐ addition and multiplication $\quad$ ☐ addition and division

☐ subtraction and multiplication $\quad$ ☐ subtraction and division

☐ addition and subtraction $\quad$ ☐ multiplication and division

Mathematicians have ordered the operations to avoid writing brackets all the time. The order is:

1. Operations in brackets.
2. Multiplication and division, from left to right.
3. Addition and subtraction, from left to right

Example: $3 \times 5 + 3 \times 6 = (3 \times 5) + (3 \times 6)$
$= 15 + 18$
$= 33$

but

$3 \times (5 + 3) \times 6$
$= 3 \times 8 \times 6$
$= 144$

Notice that the brackets in the first expression are not necessary.

7. Evaluate each expression. Use the correct order of operations.

a) $4 \times 2 - 7$ b) $3 + 6 \div 3$ c) $9 - 2 \times 4$ d) $70 \div 7 + 4$

e) $9 + 9 \div 3 - 5$ f) $3 \times 7 - 6 \div 2$ g) $(9 - 5) \times 3$ h) $(17 - 9) \div 4$

8. Translate the instructions into mathematical expressions.

a) Add 8 and 3. Then subtract 4. Then multiply by 3. *$(8 + 3 - 4) \times 3$*

b) Subtract 6 from 9. Then multiply by 2. Then add 4. _____

c) Multiply 6 and 5. Then subtract from 32. Then add 5. _____

9. Write the expressions in words.

a) $(6 + 2) \times 3$ Add and . Then multiply by .

b) $(24 - 2 \times 6) \div 4$ Multiply and . Then subtract from . Then

c) $4 \times (3 - 1 + 5)$ _____

d) $(3 + 2 \times 6) \div 5$ _____

10. Calculate the expression in the box in your notebook. Which expression without brackets gives the same answer?

a) $\boxed{8 - (5 + 2)} = 8 - 5 - 2$ or $8 - 5 + 2$ b) $\boxed{7 - (3 - 2)} = 7 - 3 - 2$ or $7 - 3 + 2$

c) $\boxed{7 + (5 - 2)} = 7 + 5 - 2$ or $7 + 5 + 2$ d) $\boxed{6 + (2 + 4)} = 6 + 2 + 4$ or $6 + 2 - 4$

11. a) Add brackets in different ways to get as many different answers as you can.

i) $3 + 1 \times 7 - 2$ ii) $16 - 4 \times 2 + 8$ iii) $16 \div 4 \times 2 + 8$

b) How many different answers did you get in part a)? i) _____ ii) _____ iii) _____

12. Rewrite each expression without brackets by changing only operations symbols. Keep the answer the same.

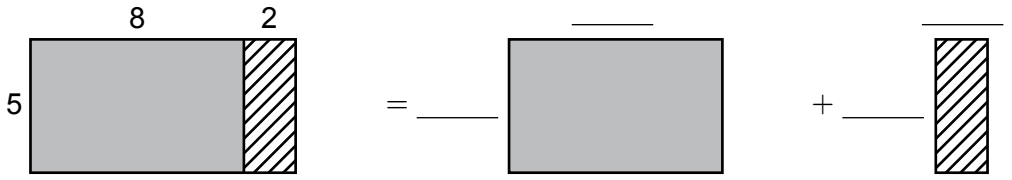
a) $5 \times 8 \div (4 \div 2)$ b) $5 \times 8 \div (4 \times 2)$ c) $5 \times 8 \times (4 \div 2)$

INVESTIGATION 1 ▶ What types of expressions can be written without brackets?

A. Write the dimensions of the two smaller rectangles in the blanks.

Write the area of the large rectangle in two ways: one with brackets and one without.

Then write an equation.



Area $= 5 \times$ (_____ $+$ _____) and Area $=$ _____ $\times$ _____ $+$ _____ $\times$ _____

So $5 \times$ (_____ $+$ _____) $=$ _____ $\times$ _____ $+$ _____ $\times$ _____

B. Write these expressions without brackets. Draw a picture if it helps.

a) $7 \times (10 + 2) =$ _____ $\times$ _____ $+$ _____ $\times$ _____

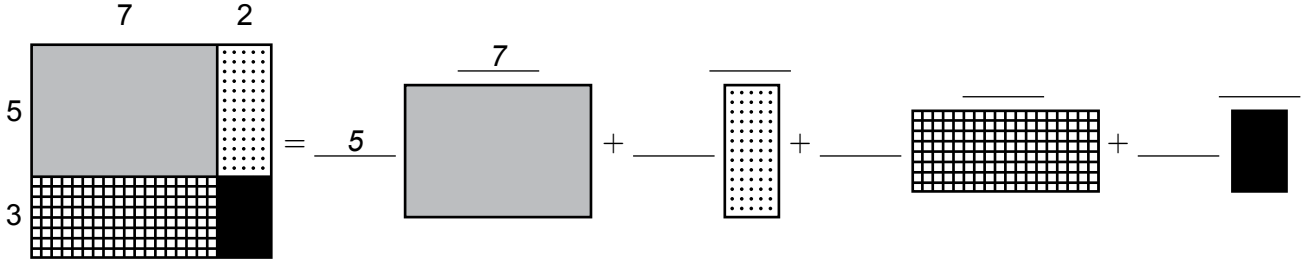
b) $4 \times (20 + 3) =$ _____ $\times$ _____ $+$ _____ $\times$ _____

C. Use your answers in B. to find 7×12 and 4×23.

D. Write the dimensions of the four smaller rectangles in the blanks.

Write the area of the large rectangle in two ways, one with brackets and one without.

Then write an equation.



Area $=$ (*5* $+$ *3*) $\times$ (_____ $+$ _____) and

Area $=$ *5* $\times$ *7* $+$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____

So $(5 + 3) \times (7 + 2) =$ _____.

E. Write these expressions without brackets. Draw pictures in your notebook to show your answers.

a) $(10 + 2) \times (10 + 3) =$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____

b) $(20 + 7) \times (40 + 5) =$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____

c) $(3 + 4 + 5) \times 2 =$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____

d) $2 \times (3 + 4 + 5) =$ _____ $\times$ _____ $+$ _____ $\times$ _____ $+$ _____ $\times$ _____

F. Use your answers to part E to find 12×13 and 27×45.

G. Calculate both sides to determine which equations are true.

a) $6 \div (2 + 1) = 6 \div 2 + 6 \div 1$

$6 \div (2 + 1) =$ _____ $\div$ _____ $=$ _____

$6 \div 2 + 6 \div 1 =$ _____ $+$ _____ $=$ _____

Is the equation true? _____

b) $(6 + 4) \div 2 = 6 \div 2 + 4 \div 2$

$(6 + 4) \div 2 =$ _____ $\div$ _____ $=$ _____

$6 \div 2 + 4 \div 2 =$ _____ $+$ _____ $=$ _____

Is the equation true? _____

c) $(5 - 2) \times 4 = 5 \times 4 - 2 \times 4$

$(5 - 2) \times 4 =$ _____ $\times$_____ $=$ _____

$5 \times 4 - 2 \times 4 =$ _____ $-$ _____ $=$ _____

Is the equation true? _____

d) $(8 - 2) \times (8 - 3) = 8 - 2 \times 3$

$(8 - 2) \times (8 - 3) =$ _____ $\times$ _____ $=$ _____

$8 - 2 \times 3 =$ _____ $-$ _____ $=$ _____

Is the equation true? _____

e) $10 \div (4 - 2) = 10 \div 4 - 10 \div 2$

$10 \div (4 - 2) =$ _____ $\div$ _____ $=$ _____

$10 \div 4 - 10 \div 2 =$ _____ $-$ _____ $=$ _____

Is the equation true? _____

f) $5 \times (8 - 3) = 5 \times 8 - 5 \times 3$

$5 \times (8 - 3) =$ _____ $\times$ _____ $=$ _____

$5 \times 8 - 5 \times 3 =$ _____ $-$ _____ $=$ _____

Is the equation true? _____

H. Match each expression with its description.

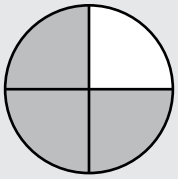
$8 \div (4 + 3)$	Can be written without brackets and does not require writing more numbers.
$8 - (4 + 3)$	Can be written without brackets but requires writing more numbers.
$8 \times (4 + 3)$	Cannot be written without brackets (using only $+$, $-$, $\times$, $\div$).

NS8-8 Fractions

Fractions name equal parts of a whole.

This pie is cut into 4 equal parts, and 3 of the parts are shaded.

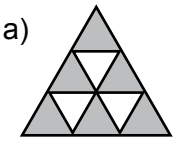
So $\frac{3}{4}$ of the pie is shaded.

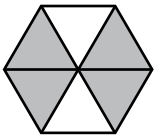


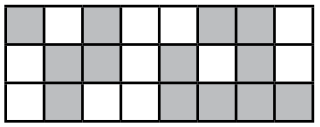
$\frac{3}{4}$ ← The **numerator** tells you how many parts are counted.

$\frac{3}{4}$ ← The **denominator** tells you how many equal parts are in a whole.

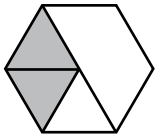
1. Name the following fractions.

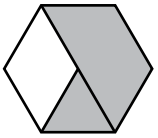
a) 

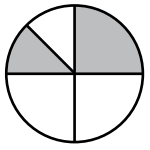
b) 

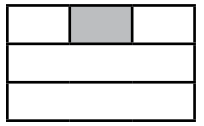
c) 

2. Draw lines to divide each figure into equal parts. Then say what fraction of each figure is shaded.

a) 

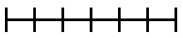
b) 

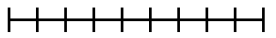
c) 

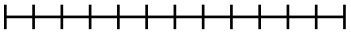
d) 

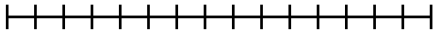
3. Divide each line into the given parts, as done in part a).

a) Thirds 

b) Halves 

c) Thirds 

d) Quarters 

e) Fifths 

Fractions can name parts of a set. In this set, $\frac{3}{5}$ of the figures are pentagons, $\frac{1}{5}$ are squares, and $\frac{1}{5}$ are circles:

4. Fill in the blanks for this set:

a) $\frac{4}{10}$ of the figures are _____.

b) $\frac{3}{10}$ of the figures are _____.

c) _____ of the figures are squares.

d) _____ of the figures are triangles.

5.

	Whole Numbers from 2 to 9	**Whole Numbers from 10 to 16**
Prime Numbers	*2, 3,*	
Composite Numbers		

a) Fill in the chart.

b) What fraction of the whole numbers from 2 to 9 are composite?

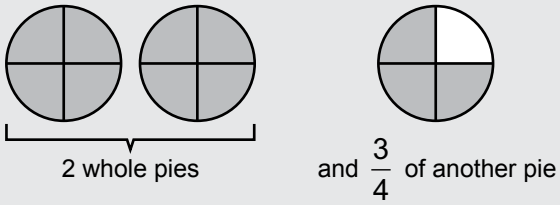
c) What fraction of the whole numbers from 2 to 16 are prime?

NS8-9 Mixed Numbers

Mattias and his friends ate the amount of pie shown.

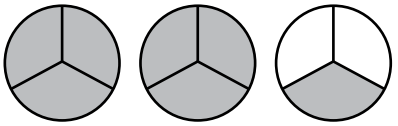
They ate two and three quarter pies altogether (or $2\frac{3}{4}$ pies).

$2\frac{3}{4}$ is called a **mixed number** because it is a mixture of a whole number and a fraction.



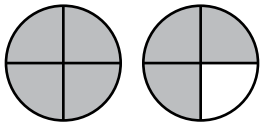
1. Follow the example to find the **mixed number** for each picture.

a)



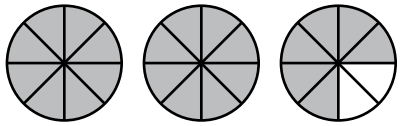
__2__ whole pies and __$\frac{1}{3}$__ of another pie $= 2\frac{1}{3}$ pies

b)



____ whole pie and ____ of another pie = ____ pies

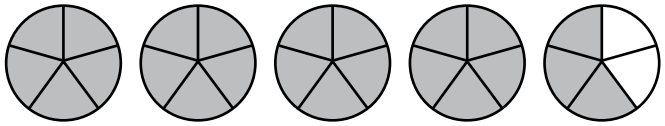
c)



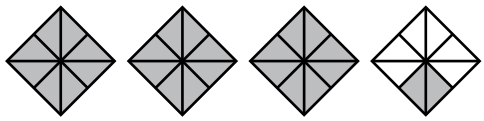
____ whole pies and ____ of another pie = ____ pies

2. Write each fraction as a **mixed number**.

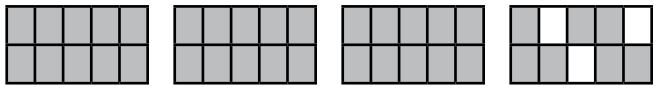
a)



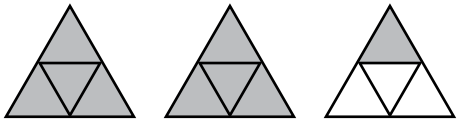
b)



c)

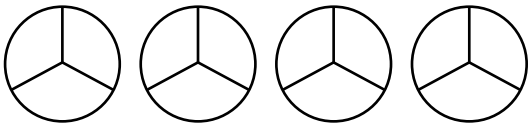


d)

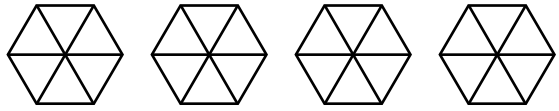


3. Shade the area given by the mixed number. Note: There may be more figures than you need.

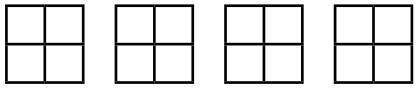
a) $2\frac{2}{3}$



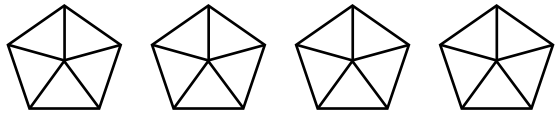
b) $3\frac{5}{6}$



c) $1\frac{3}{4}$



d) $2\frac{4}{5}$



4. Sketch:

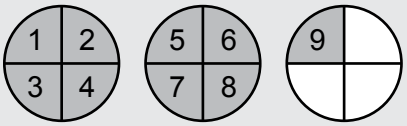
a) $2\frac{1}{3}$ pies b) $3\frac{3}{4}$ squares c) $1\frac{3}{5}$ pies d) $2\frac{5}{6}$ rectangles e) $3\frac{7}{8}$ circles

5. Order from smallest to largest: $4\frac{2}{3}$, $4\frac{1}{4}$, $3\frac{3}{4}$.

6. Which is closer to 5: $5\frac{3}{4}$ or $4\frac{2}{3}$? Explain.

NS8-10 Improper Fractions

Huan-Yue and her friends ate **9** quarter-sized pieces of pizza:



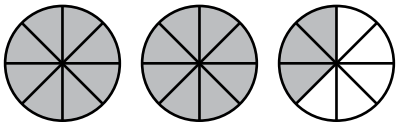
Altogether, they ate $\frac{9}{4}$ pizzas.

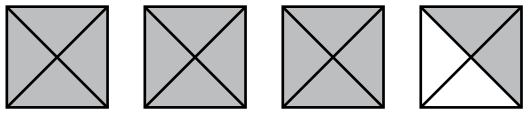
$\frac{9}{4} = 2\frac{1}{4}$

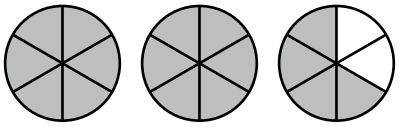
improper fraction — mixed number

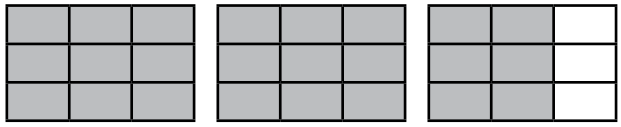
When the numerator of a fraction is larger than the denominator, the fraction represents **more than a whole**. Such fractions are called **improper fractions**.

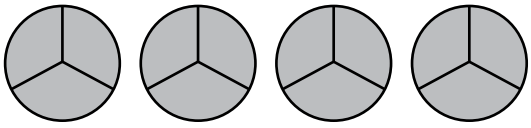
1. Write these fractions as **improper fractions**.

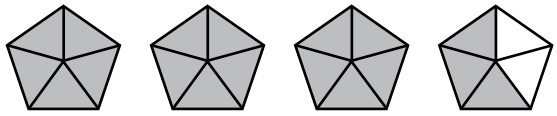
a) 

b) 

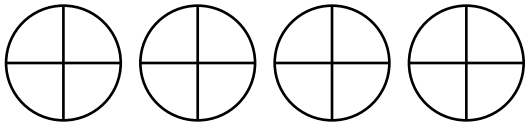
c) 

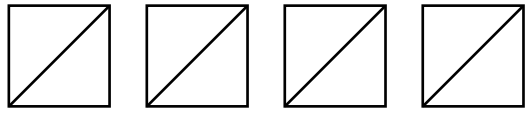
d) 

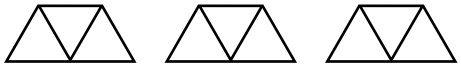
e) 

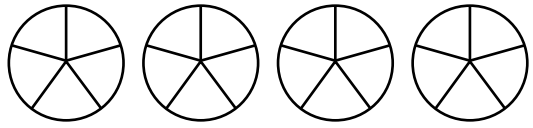
f) 

2. Shade one piece at a time until you have shaded the amount given by the improper fraction.

a) $\frac{13}{4}$ 

b) $\frac{5}{2}$ 

c) $\frac{8}{3}$ 

d) $\frac{15}{5}$ 

3. Sketch:

a) $\frac{7}{3}$ pies

b) $\frac{13}{4}$ squares

c) $\frac{9}{2}$ parallelograms

d) $\frac{11}{6}$ rectangles

e) $\frac{17}{8}$ circles

4. Order from smallest to largest: $\frac{7}{4}, \frac{9}{4}, \frac{9}{3}, \frac{10}{3}$.

5. Which fractions are improper fractions? How do you know?

a) $\frac{5}{7}$

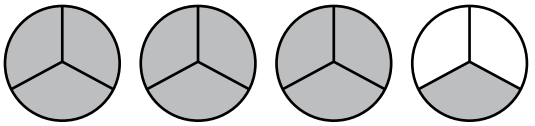
b) $\frac{13}{11}$

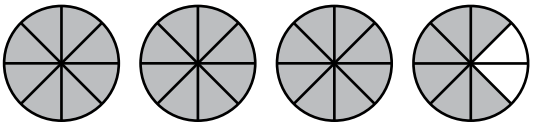
c) $1\frac{9}{8}$

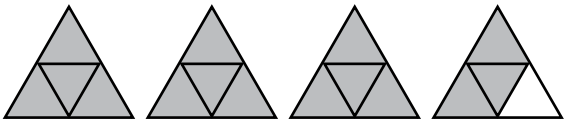
d) $\frac{8}{3}$

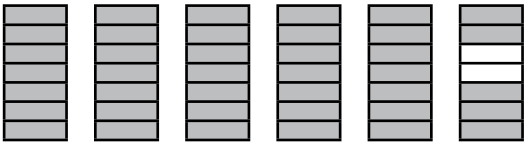
NS8-11 Mixed and Improper Fractions

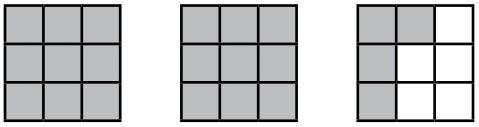
1. Write these fractions as **mixed numbers** and as **improper fractions**.

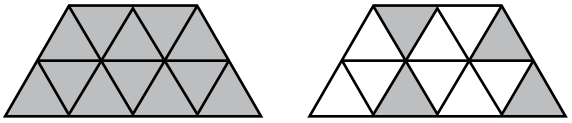
a) 

b) 

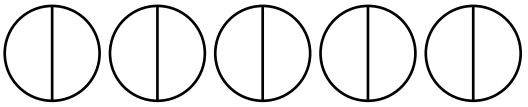
c) 

d) 

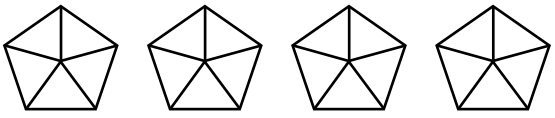
e) 

f) 

2. Shade the amount of pie given by the mixed fraction. Then write an improper fraction for the amount.

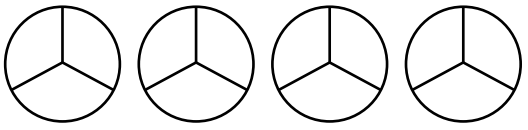
a) $4\frac{1}{2}$ 

Improper fraction: _____

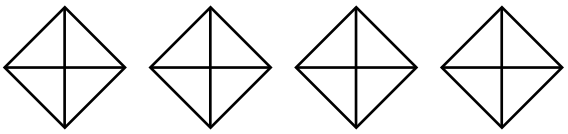
b) $3\frac{3}{5}$ 

Improper fraction: _____

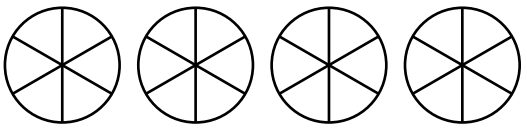
3. Shade the amount of area given by the improper fraction. Then write a mixed number for the amount.

a) $\frac{11}{3}$ 

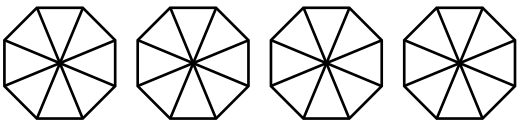
Mixed number: _____

b) $\frac{11}{4}$ 

Mixed number: _____

c) $\frac{17}{6}$ 

Mixed number: _____

d) $\frac{21}{8}$ 

Mixed number: _____

4. Draw a picture to find out which fraction is greater.

a) $3\frac{1}{2}, 2\frac{2}{3}, \frac{5}{3}$

b) $1\frac{4}{5}, 2\frac{1}{4}, \frac{11}{5}$

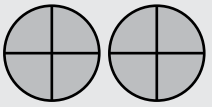
c) $\frac{13}{4}, \frac{7}{2}, 2\frac{2}{3}$

d) $\frac{15}{8}, \frac{13}{5}, \frac{7}{3}$

5. How could you use division to find out how many whole pies are in $\frac{24}{7}$ of a pie? Explain.

NS8-12 More Mixed Numbers

There are 4 quarter pieces in 1 pie.



There are 8 (2×4) quarters in 2 pies.

There are 12 (3×4) quarters in 3 pies.

How many quarter pieces are in $3\frac{1}{4}$ pies?

12 pieces (3×4) + 1 extra piece → $3\frac{1}{4}$

So there are 13 quarter pieces altogether.

1. Find the number of **halves** in each amount.

a) 1 pie = _____ halves

b) 3 pies = _____ halves

c) 5 pies = _____ halves

d) $2\frac{1}{2}$ pies = _____ halves

e) $4\frac{1}{2}$ pies = _____ halves

f) $6\frac{1}{2}$ pies = _____

2. Find the number of **thirds** in each amount.

a) 1 pie = _____ thirds

b) 2 pies = _____ thirds

c) 3 pies = _____ thirds

d) $1\frac{1}{3}$ pies = _____ thirds

e) $3\frac{2}{3}$ pies = _____

f) $5\frac{1}{3}$ pies = _____

3. A box holds 4 cans, so each can is a **fourth**. Find the number of cans in each amount.

a) 2 boxes hold _____ cans

b) $2\frac{1}{2}$ boxes hold _____ cans

c) $4\frac{3}{4}$ boxes hold _____ cans

4. If a bag holds 16 peas, then…

a) $1\frac{1}{16}$ bags hold _____ peas

b) $2\frac{1}{2}$ bags hold _____ peas

c) $3\frac{1}{4}$ bags hold _____ peas

5. Write the mixed numbers as improper fractions.

a) $2\frac{2}{3} = \frac{\quad}{3}$

b) $3\frac{1}{2} = \frac{\quad}{2}$

c) $5\frac{4}{5} =$

d) $4\frac{3}{4} =$

e) $5\frac{2}{7} =$

6. Envelopes come in packs of 8. Alice used $3\frac{7}{8}$ packs. How many envelopes did she use? _____

7. Maia and her friends ate $2\frac{3}{4}$ pizzas. How many quarter-sized pieces did they eat? _____

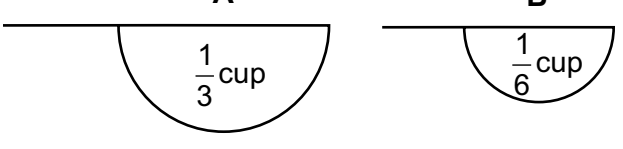
BONUS ▶

8. How many quarters are there in $7\frac{1}{2}$ dollars? _____

9. Cindy needs $4\frac{2}{3}$ cups of flour.

a) How many scoops of cup A would she need? _____

b) How many scoops of cup B would she need? _____



NS8-13 More Mixed Numbers and Improper Fractions

How many whole pies are there in $\frac{13}{4}$ pies?

There are 13 pieces altogether, and each pie has 4 pieces. So you can find the number of whole pies by dividing 13 by 4: **13 ÷ 4 = 3 remainder 1**

There are 3 whole pies and 1 quarter left over: $\frac{13}{4} = 3\frac{1}{4}$

1. Find the number of whole pies in each amount by dividing.

a) $\frac{4}{2}$ pies = _____ whole pies b) $\frac{15}{3}$ pies = _____ whole pies c) $\frac{16}{4}$ pies = _____ whole pies

d) $\frac{21}{7}$ pies = _____ whole pies e) $\frac{25}{5}$ pies = _____ whole pies f) $\frac{30}{6}$ pies = _____ whole pies

2. Find the number of whole pies and the number of pieces remaining by dividing.

a) $\frac{5}{2}$ pies = *2* whole pies and *1* half pie = $2\frac{1}{2}$ pies

b) $\frac{11}{2}$ pies = _____ whole pies and _____ half pie = _____ pies

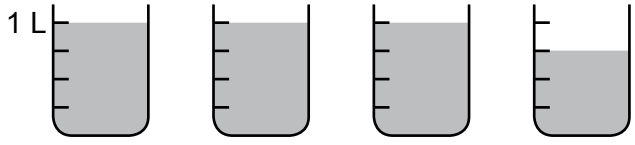
c) $\frac{13}{3}$ pies = _____ whole pies and _____ third = _____ pies

d) $\frac{17}{4}$ pies = _____ whole pies and _____ fourth = _____ pies

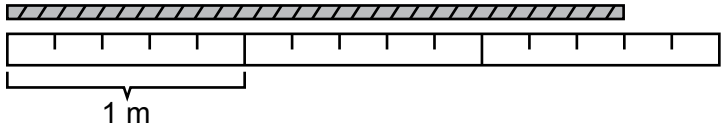
3. Write the following improper fractions as mixed numbers.

a) $\frac{5}{2}$ b) $\frac{14}{3}$ c) $\frac{17}{6}$ d) $\frac{21}{4}$ e) $\frac{29}{5}$ f) $\frac{31}{7}$ g) $\frac{70}{9}$ h) $\frac{61}{8}$

4. Write a mixed number and improper fraction for the total number of litres:



5. Write a mixed number and improper fraction for the length of the rope:



6. Order from smallest to largest: $\frac{7}{3}$, $\frac{9}{4}$, $\frac{5}{2}$.

7. Between which two whole numbers is $\frac{21}{8}$?

8. How much greater than a whole is each fraction?

a) $\frac{11}{7}$ b) $\frac{8}{5}$ c) $\frac{5}{3}$ d) $\frac{19}{10}$

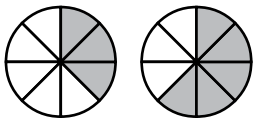
9. Which fractions are greater than 3 but less than 4?

a) $\frac{17}{4}$ b) $\frac{5}{3}$ c) $\frac{16}{5}$ d) $\frac{5}{2}$ e) $\frac{11}{6}$

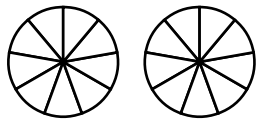
NS8-14 Comparing Fractions — Introduction

1. Shade the given amount in each pie. Then circle the greater fraction in each pair.

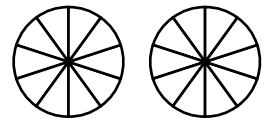
a) $\frac{3}{8}$ $\frac{5}{8}$



b) $\frac{7}{9}$ $\frac{5}{9}$



c) $\frac{9}{10}$ $\frac{8}{10}$

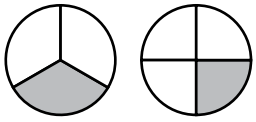


2. Two fractions have the same denominators (bottoms) but different numerators (tops)?

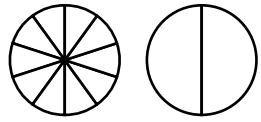
How can you tell which fraction is greater?

3. Shade the given amount in each pie. Then circle the greater fraction in each pair.

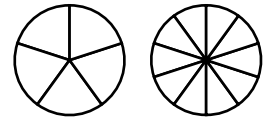
a) $\frac{1}{3}$ $\frac{1}{4}$



b) $\frac{1}{10}$ $\frac{1}{2}$



c) $\frac{3}{5}$ $\frac{3}{10}$



4. Two fractions have the same numerators (tops) but different denominators (bottoms).

How can you tell which fraction is greater?

5. Write the fractions in order from least to greatest.

a) $\frac{1}{8}, \frac{1}{3}, \frac{1}{15}$

____ ____ ____

b) $\frac{2}{9}, \frac{2}{6}, \frac{2}{8}, \frac{2}{12}$

____ ____ ____ ____

c) $\frac{4}{5}, \frac{1}{5}, \frac{3}{5}$

____ ____ ____

d) $\frac{9}{10}, \frac{2}{10}, \frac{1}{10}, \frac{5}{10}$

____ ____ ____ ____

e) $\frac{5}{8}, \frac{7}{8}, \frac{5}{9}$

____ ____ ____

f) $\frac{4}{7}, \frac{3}{7}, \frac{4}{5}$

____ ____ ____

BONUS ▶ $\frac{15}{19}$ $\frac{9}{23}$ $\frac{11}{21}$ $\frac{11}{19}$ $\frac{6}{23}$ $\frac{9}{22}$ $\frac{15}{17}$ $\frac{9}{21}$

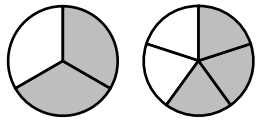
6. Which fraction is greater? How do you know?

a) $\frac{7}{5}$ or $\frac{9}{5}$

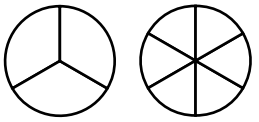
b) $4\frac{1}{4}$ or $4\frac{3}{4}$

NS8-15 Equivalent Fractions

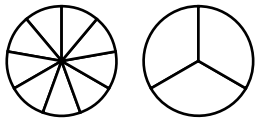
1. Compare the fractions by shading to see which is more. Write > (more than), < (less than), or = (equal).

a) 

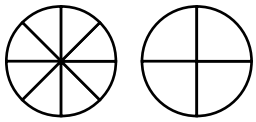
$\frac{2}{3}$ > $\frac{3}{5}$

b) 

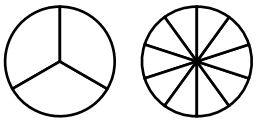
$\frac{2}{3}$ ☐ $\frac{4}{6}$

c) 

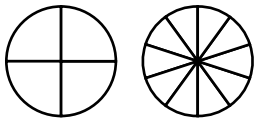
$\frac{5}{9}$ ☐ $\frac{2}{3}$

d) 

$\frac{6}{8}$ ☐ $\frac{3}{4}$

e) 

$\frac{2}{3}$ ☐ $\frac{7}{10}$

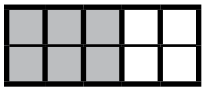
f) 

$\frac{3}{4}$ ☐ $\frac{6}{10}$

Two fractions are said to be **equivalent** if they represent the same amount.

2. List two pairs of equivalent fractions from Question 1. _____ = _____ and _____ = _____

3. Group the squares to make an equivalent fraction. How many of the equal larger groups are shaded?

a) 

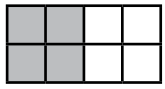
$\frac{6}{10} = \frac{3}{5}$

b) 

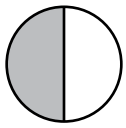
$\frac{4}{6} = \frac{\_}{3}$

c) 

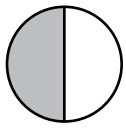
$\frac{10}{12} = \frac{\_}{6}$

4. Write three equivalent fractions for the amount shaded here:  _____ _____ _____

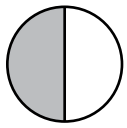
5. a) Draw lines to cut the pies into:



4 equal pieces



6 equal pieces

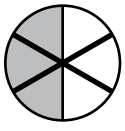


8 equal pieces

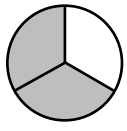
b) Then fill in the numerators of the equivalent fractions: $\frac{1}{2} = \frac{\_}{4} = \frac{\_}{6} = \frac{\_}{8}$

6. Make an equivalent fraction by cutting each piece into the same number of parts.

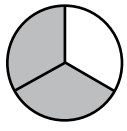
a) $\frac{1}{2} = \frac{3}{6}$



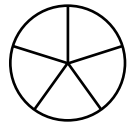
b) $\frac{2}{3} = \frac{4}{\_}$



c) $\frac{2}{3} = \frac{\_}{9}$



d) $\frac{2}{5} = \frac{\_\_}{15}$



NS8-16 Comparing Fractions Using Equivalent Fractions

1. Write six equivalent fractions by skip counting to find the numerators.

a) $\frac{2}{3} = \frac{\_}{6} = \frac{\_}{9} = \frac{\_\_}{12} = \frac{\_\_}{15} = \frac{\_\_}{18} = \frac{\_\_}{21}$

b) $\frac{3}{5} = \frac{\_\_}{10} = \frac{\_\_}{15} = \frac{\_\_}{20} = \frac{\_\_}{25} = \frac{\_\_}{30} = \frac{\_\_}{35}$

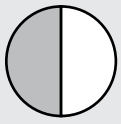
2. Find two fractions with the same denominators from the lists in Question 1. _____ and _____

Which fraction is greater: $\frac{2}{3}$ or $\frac{3}{5}$? _____

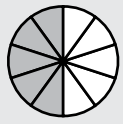
How do you know? _____

When you multiply the numerator and denominator of a fraction by the same number, you create an **equivalent fraction**.

$$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$



You are cutting each piece into 5 parts



3. Create an equivalent fraction with denominator 36 by multiplying the numerator and denominator by the same number:

a) $\frac{1}{2} \begin{matrix} \times 18 \\ \times 18 \end{matrix} = \frac{18}{36}$ b) $\frac{4}{9} = \frac{\_\_}{36}$ c) $\frac{5}{6} = \frac{\_\_}{36}$ d) $\frac{11}{18} = \frac{\_\_}{36}$

e) $\frac{2}{3} = \frac{\_\_}{36}$ f) $\frac{3}{4} = \frac{\_\_}{36}$ g) $\frac{1}{6} = \frac{\_\_}{36}$ h) $\frac{5}{12} = \frac{\_\_}{36}$

4. Write the fractions from Question 3 in order from smallest to largest.

5. a) Write several fractions equivalent to $\frac{1}{2}$.

$\frac{1}{2} = \frac{\_}{4} = \frac{\_}{6} = \frac{\_}{8} = \frac{\_\_}{10} = \frac{\_\_}{12} = \frac{\_\_}{14} = \frac{\_\_}{16} = \frac{\_\_}{18} = \frac{\_\_}{20}$

b) How much more than a half is each fraction below?

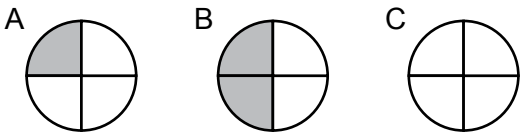
$\frac{3}{4}$ is _____ more than $\frac{1}{2}$ $\frac{4}{6}$ is _____ more than $\frac{1}{2}$ $\frac{5}{8}$ is _____ more than $\frac{1}{2}$

$\frac{6}{10}$ is _____ more than $\frac{1}{2}$ $\frac{7}{12}$ is _____ more than $\frac{1}{2}$ $\frac{8}{14}$ is _____ more than $\frac{1}{2}$

c) Write all the given fractions from part b) in order from smallest to largest.

NS8-17 Adding and Subtracting Fractions — Introduction

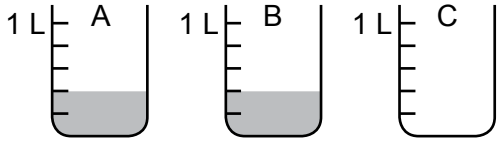
1. Imagine moving the shaded pieces from pies A and B into pie plate C.
Show how much of pie C would be filled, then write a fraction for pie C.



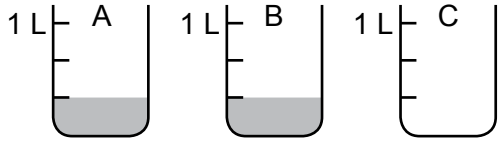
$\frac{1}{4} + \frac{2}{4} = \_\_\_\_$

2. Imagine pouring the liquid from cups A and B into cup C.

Shade the amount of liquid that would be in C. Then complete the addition statements.



$\frac{\ }{5} + \frac{\ }{5} = \_\_\_\_$



$\frac{\ }{3} + \frac{\ }{3} = \_\_\_\_$

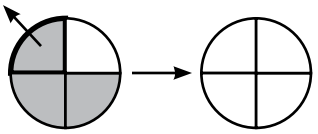
3. Add.

a) $\frac{3}{5} + \frac{1}{5} =$ b) $\frac{1}{4} + \frac{2}{4} =$ c) $\frac{2}{7} + \frac{4}{7} =$ d) $\frac{5}{8} + \frac{2}{8} =$

e) $\frac{3}{11} + \frac{6}{11} =$ f) $\frac{10}{17} + \frac{6}{17} =$ g) $\frac{15}{24} + \frac{4}{24} =$ h) $\frac{18}{57} + \frac{13}{57} =$

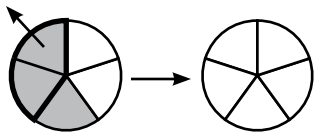
4. Show how much pie would be left if you took away the amount shown.
Then complete the fraction statement.

a)



$\frac{3}{4} - \frac{1}{4} = \_\_\_\_$

b)



$\frac{3}{5} - \frac{2}{5} = \_\_\_\_$

5. Subtract.

a) $\frac{2}{3} - \frac{1}{3} =$ b) $\frac{3}{5} - \frac{2}{5} =$ c) $\frac{6}{7} - \frac{3}{7} =$ d) $\frac{5}{8} - \frac{2}{8} =$

e) $\frac{10}{12} - \frac{3}{12} =$ f) $\frac{6}{19} - \frac{4}{19} =$ g) $\frac{9}{28} - \frac{3}{28} =$ h) $\frac{17}{57} - \frac{12}{57} =$

6. Calculate.

a) $\frac{2}{7} + \frac{1}{7} + \frac{3}{7} =$ b) $\frac{4}{11} + \frac{5}{11} - \frac{2}{11} =$ c) $\frac{10}{18} - \frac{7}{18} + \frac{5}{18} =$

NS8-18 Adding and Subtracting Fractions

To add fractions with different denominators:

Step 1: Find the LCM of the denominators.

$\frac{1}{3}+\frac{2}{5}$

Multiples of 3: 0, 3, 6, 9, 12, **15**, 18

Multiples of 5: 0, 5, 10, **15**, 20, 25, 30

LCM (3, 5) = 15

Step 2: Create equivalent fractions with that denominator.

$$\frac{1}{3}+\frac{2}{5} = \begin{matrix} 5\times \\ 5\times \end{matrix}\frac{1}{3}+\frac{2}{5}\begin{matrix} \times 3 \\ \times 3 \end{matrix}$$

$$= \frac{5}{15}+\frac{6}{15}$$

$$= \frac{11}{15}$$

The LCM of the denominators is called the **lowest common denominator (LCD)** of the fractions.

1. Find the LCD of each pair of fractions. Then show what numbers you would multiply the numerator and denominator of each fraction by in order to add.

a) LCD = *6*

$\begin{matrix} 3\times \\ 3\times \end{matrix}\frac{1}{2}+\frac{2}{3}\begin{matrix} \times 2 \\ \times 2 \end{matrix}$

b) LCD = _____

$\frac{3}{4}+\frac{1}{8}$

c) LCD = _____

$\frac{1}{30}+\frac{1}{6}$

d) LCD = _____

$\frac{3}{4}+\frac{2}{3}$

e) LCD = _____

$\frac{3}{7}+\frac{1}{3}$

f) LCD = _____

$\frac{3}{4}+\frac{1}{6}$

g) LCD = _____

$\frac{4}{5}+\frac{1}{10}$

h) LCD = _____

$\frac{1}{8}+\frac{5}{7}$

2. Add or subtract the fractions by changing them to equivalent fractions with denominator equal to the LCD of the fractions.

a) $\frac{2}{5}+\frac{1}{4}$

$=$

$=$

b) $\frac{4}{15}+\frac{2}{3}$

$=$

$=$

c) $\frac{2}{3}-\frac{1}{8}$

$=$

$=$

d) $\frac{2}{3}-\frac{1}{12}$

$=$

$=$

e) $\frac{3}{4}+\frac{1}{8}$

f) $\frac{1}{6}+\frac{13}{24}$

g) $\frac{11}{28}-\frac{2}{7}$

h) $\frac{4}{7}+\frac{1}{8}$

i) $\frac{4}{9}-\frac{1}{6}$

3. Add or subtract.

a) $\frac{5}{6}+\frac{1}{12}$

b) $\frac{19}{25}-\frac{3}{5}$

c) $\frac{5}{7}-\frac{1}{4}$

d) $\frac{4}{9}+\frac{2}{5}$

e) $\frac{5}{8}-\frac{7}{12}$

f) $\frac{2}{3}+\frac{1}{4}+\frac{1}{2}$

g) $\frac{3}{15}+\frac{2}{3}+\frac{1}{5}$

h) $\frac{11}{15}+\frac{2}{3}-\frac{1}{5}$

i) $\frac{3}{5}+\frac{17}{30}-\frac{5}{6}$

A fraction is reduced to **lowest terms** when the greatest common factor of its numerator and denominator is the number 1.

$\frac{6}{8}$ **is not** in lowest terms because the GCF of 6 and 8 is 2.

Factors of 6: 1, **2**, 3, 6
Factors of 8: 1, **2**, 4, 8

$\frac{3}{4}$ **is** in lowest terms because the GCF of 3 and 4 is 1.

Factors of 3: **1**, 3
Factors of 4: **1**, 2, 4

4. Find the GCF of the numerator and denominator. Is the fraction in lowest terms? Write yes or no.

a) $\frac{2}{6}$ GCF = *2* — *no*

b) $\frac{3}{5}$ GCF = _____ _____

c) $\frac{4}{5}$ GCF = _____ _____

d) $\frac{5}{10}$ GCF = _____ _____

e) $\frac{8}{10}$ GCF = _____ _____

f) $\frac{7}{10}$ g) $\frac{15}{16}$ h) $\frac{14}{12}$ i) $\frac{9}{5}$ j) $\frac{5}{9}$

To reduce a fraction to lowest terms:

Step 1: Find the GCF of the numerator and denominator

Step 2: Divide both the numerator and denominator by the GCF.

5. Reduce the fractions below by dividing the numerator and the denominator by their GCF.

a) $\frac{2 \div 2}{10 \div 2} = \frac{1}{5}$ b) $\frac{2 \div}{6 \div} =$ _____ c) $\frac{2 \div}{8 \div} =$ _____ d) $\frac{2 \div}{12 \div} =$ _____

e) $\frac{6}{9} =$ _____ f) $\frac{3}{15} =$ _____ g) $\frac{4}{12} =$ _____ h) $\frac{20}{25} =$ _____

6. Add or subtract, then reduce your answer to lowest terms.

a) $\frac{5 \times 1}{5 \times 6} + \frac{1 \times 3}{10 \times 3}$

$= \frac{5}{30} + \frac{3}{30}$

$= \frac{8}{30} = \frac{4}{15}$

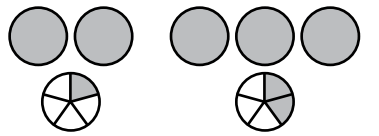
b) $\frac{13}{15} - \frac{2}{5}$ c) $\frac{5}{6} + \frac{7}{10}$ d) $\frac{22}{28} - \frac{2}{7}$

e) $\frac{1}{10} + \frac{1}{2} + \frac{1}{5}$ f) $\frac{5}{8} + \frac{1}{5} + \frac{1}{20}$ g) $\frac{1}{7} + \frac{4}{5} - \frac{8}{35}$ h) $\frac{5}{7} - \frac{8}{21} + \frac{2}{3}$

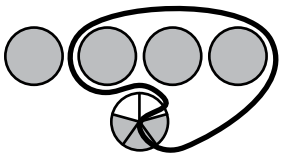
NS8-19 Adding and Subtracting Mixed Numbers

1. Add or subtract.

a) $2\frac{1}{5} + 3\frac{2}{5} =$ _____



b) $4\frac{3}{5} - 3\frac{1}{5} =$



c) $2\frac{1}{5} + 2\frac{2}{5} =$

d) $3\frac{3}{7} + 2\frac{1}{7} =$

e) $5\frac{7}{8} - 3\frac{2}{8} =$

f) $7\frac{9}{15} - 4\frac{2}{15} =$

2. Add or subtract by changing the fractions to equivalent fractions.

a) $2\frac{1}{2} + 1\frac{1}{3}$

$= 2 + 1 + \frac{1}{2} + \frac{1}{3}$

$= 3 + \frac{\ }{6} + \frac{\ }{6}$

$= 3\frac{\ }{6}$

b) $3\frac{3}{4} - 1\frac{1}{3}$

$= 3 - 1 + \frac{3}{4} - \frac{1}{3}$

$= 2 + \frac{\ }{12} - \frac{\ }{12}$

$= 2\frac{\ }{12}$

c) $5\frac{2}{3} - 2\frac{3}{5}$

d) $2\frac{2}{7} + 4\frac{1}{2}$

e) $4\frac{2}{5} - 1\frac{1}{6}$

f) $2\frac{3}{8} + 4\frac{1}{3}$

3. $1\frac{1}{2} + 2\frac{2}{3} = 3\frac{7}{6}$. How can you simplify this answer?

4. $\frac{4}{5}$ is greater than $\frac{1}{3}$. How can you subtract $4\frac{1}{3} - 2\frac{4}{5}$?

5. a) Change the improper fractions to mixed numbers.

i) $\frac{7}{6} = 1\frac{1}{6}$

ii) $\frac{11}{5} =$

iii) $\frac{13}{7} =$

iv) $\frac{11}{4} =$

b) Rewrite each mixed number to make the improper fraction a proper fraction.

i) $3\frac{7}{6} = 3 + \frac{7}{6}$

$= 3 + 1\frac{1}{6}$

$= 4\frac{1}{6}$

ii) $2\frac{4}{3} =$

$=$

$=$

iii) $4\frac{8}{5} =$

$=$

$=$

c) Add by changing the fractions to equivalent fractions. Simplify your answer as in part b).

i) $2\frac{2}{5}+\frac{2}{3}$

$= 2 + \frac{2}{5} + \frac{2}{3}$

$= 2 + \frac{\quad}{15} + \frac{\quad}{15}$

$= 2\frac{\quad}{15} = 3\frac{\quad}{15}$

ii) $3\frac{2}{3}+\frac{5}{6}$

iii) $4\frac{3}{4}+2\frac{3}{5}$

6. a) Rewrite each mixed number by regrouping 1 whole as a fraction.

Example: $4\frac{1}{3} = 3 + 1\frac{1}{3} = 3\frac{4}{3}$

i) $5\frac{3}{4}$ ii) $5\frac{1}{2}$ iii) $1\frac{1}{6}$ iv) $2\frac{3}{4}$ v) $3\frac{2}{5}$ vi) $4\frac{5}{7}$

b) Subtract by rewriting the first mixed number as in part a):

i) $3\frac{1}{5} - 1\frac{3}{4} = 3\frac{4}{20} - 1\frac{15}{20}$

$= 2\frac{24}{20} - 1\frac{15}{20} = 1\frac{9}{20}$

ii) $4\frac{1}{3}-2\frac{3}{5}$

7. Add or subtract by first changing the mixed numbers to improper fractions.

a) $3\frac{1}{3}+5\frac{3}{4}$

$=\frac{10}{3}+\frac{23}{4}$

$=\frac{40}{12}+\frac{69}{12}$

$=\frac{109}{12} = 9\frac{1}{12}$

b) $1\frac{1}{5}-\frac{2}{3}$ c) $4\frac{2}{3}+2\frac{4}{5}$ d) $5\frac{1}{8}-3\frac{1}{3}$

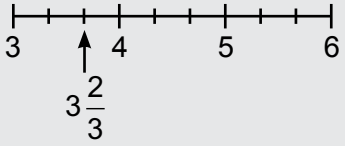
8. Sonjay cycled $6\frac{7}{8}$ km in the first hour, $5\frac{1}{2}$ km the second hour, and $4\frac{3}{4}$ km the third hour. How many kilometres did he cycle in the three hours?

9. A cafeteria sold $2\frac{5}{8}$ cheese pizzas, $4\frac{1}{3}$ vegetable pizzas, and $3\frac{1}{4}$ deluxe pizzas at lunchtime. How many pizzas did they sell altogether?

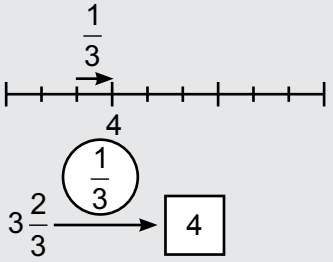
10. Gerome bought $5\frac{3}{4}$ metres of cloth. He used $3\frac{4}{5}$ to make a banner. How many metres of cloth were left over?

Sayaka subtracts $6-3\frac{2}{3}$ on a number line as follows:

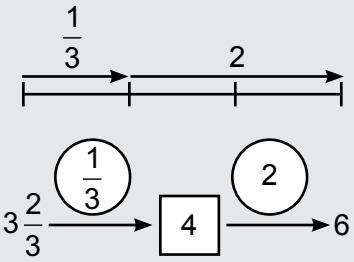
She marks the number she is subtracting $\left(3\frac{2}{3}\right)$ on a number line.



She draws an arrow to show the difference between $3\frac{2}{3}$ and the nearest whole number (4).



She draws an arrow to show the difference between 4 and 6 (= 2).

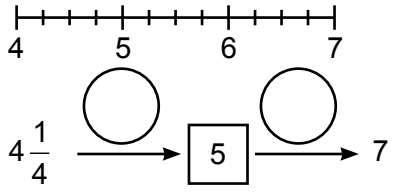


She sees that:

$$6-3\frac{2}{3}=2+\frac{1}{3}=2\frac{1}{3}$$

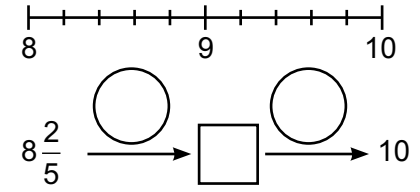
1. Follow Sayaka's steps to find the difference. The first question is started for you.

a)



$7-4\frac{1}{4}=$

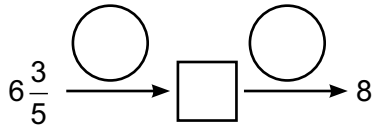
b)



$10-8\frac{2}{5}=$

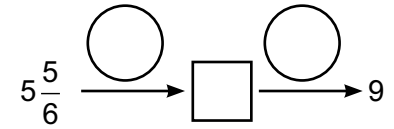
2. Find the differences.

a)



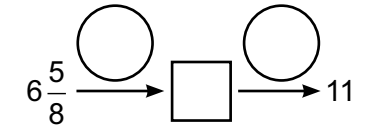
$8-6\frac{3}{5}=$

b)



$9-5\frac{5}{6}=$

c)



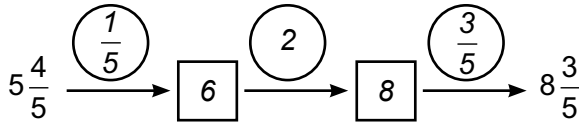
$11-6\frac{5}{8}=$

3. Find the differences mentally:

a) $4-1\frac{1}{7}=$ b) $9-6\frac{7}{9}=$ c) $12-7\frac{3}{10}=$ d) $23-20\frac{5}{8}=$

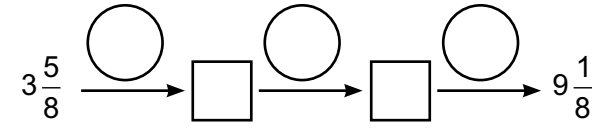
4. Find the difference by following the steps shown.

a)



$8\frac{3}{5}-5\frac{4}{5}=\frac{1}{5}+2+\frac{3}{5}=2\frac{4}{5}$

b)



5. Find the differences mentally.

a) $6\frac{2}{5}-5\frac{4}{5}=$ b) $9\frac{1}{7}-3\frac{4}{7}=$ c) $17\frac{1}{8}-13\frac{3}{8}=$ d) $13\frac{1}{6}-6\frac{5}{6}=$

NS8-21 Investigating Fractions and Division — Advanced

1. Write each improper fraction in terms of division.

a) $\frac{4}{3} =$ *4 ÷ 3* b) $\frac{7}{2} =$ _____ c) $\frac{9}{3} =$ _____ d) $\frac{5}{4} =$ _____

2. Write each mixed number in terms of addition and division.

a) $2\frac{3}{4} =$ *2 + 3 ÷ 4* b) $3\frac{2}{5} =$ _____ c) $1\frac{2}{3} =$ _____ d) $3\frac{1}{2} =$ _____

3. Which improper fraction from question 1 is equivalent to a mixed number from Question 2? Verify your answer by doing the calculations directly.

To add fractions that have a common denominator, we can add the numerators.

Example 1: $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$ Example 2: $\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$

4. Translate the examples in the box into a statement using division.

Example 1: *3 ÷ 10* + *4 ÷ 10* = *7 ÷ 10* Example 2: _____ + _____ = _____

5. Translate each statement of addition of fractions into addition of quotients. Verify by dividing and adding.

a) $\frac{9}{3} + \frac{6}{3} = \frac{9+6}{3}$ b) $\frac{8}{2} + \frac{6}{2} = \frac{8+6}{2}$ c) $\frac{20}{5} + \frac{35}{5} = \frac{20+35}{5}$

9 ÷ 3 + 6 ÷ 3 = (9 + 6) ÷ 3

3 + 2 = 15 ÷ 3

5 = 5

INVESTIGATION 1 ▶ To add fractions that have a common numerator, can we add the denominators?

A. Is $\frac{5}{8} + \frac{5}{14}$ more or less than $\frac{5}{8}$? How do you know?

B. Is $\frac{5}{8+14}$ more or less than $\frac{5}{8}$? How do you know?

C. Can $\frac{5}{8} + \frac{5}{14} = \frac{5}{8+14}$? Explain.

---

6. The symbol ≠ means "not equal to." Translate the statements using division instead of fractions. Verify by division that the two sides are not equal.

$\frac{6}{2} + \frac{6}{1} \neq \frac{6}{2+1}$ $\frac{30}{2} + \frac{30}{3} \neq \frac{30}{2+3}$ $\frac{24}{2} + \frac{24}{6} \neq \frac{24}{2+6}$ $\frac{70}{2} + \frac{70}{5} \neq \frac{70}{2+5}$

NS8-22 Word Problems with Adding and Subtracting Fractions

1. What fraction of a year is 3 months? $\frac{3}{12} = \_\_\_\_\_$

2. What fraction of an hour is 45 minutes?

3. What fraction of a metre is 20 cm?

4. A cup is 240 mL. What fraction of a cup is 100 mL?

5. Tania cycled $5\frac{3}{5}$ km in the first hour, and $4\frac{2}{3}$ km in the second hour. How many kilometres did she cycle in two hours?

6. Monica ran $6\frac{3}{8}$ km in the first hour, $4\frac{1}{2}$ km the second hour, and $4\frac{1}{4}$ km the third hour. How many kilometres did she run in three hours?

7. A cafeteria sold $2\frac{7}{8}$ cheese pizzas, $5\frac{5}{6}$ vegetable pizzas, and $7\frac{11}{12}$ deluxe pizzas at lunchtime. How many pizzas did they sell altogether?

8. Trevor has $\frac{2}{3}$ of an hour to write a test. If he finishes the test in $\frac{1}{2}$ of an hour then how much time would remain?

9. Tegan bought $1\frac{1}{6}$ metres of ribbon. She needs $\frac{2}{5}$ metres to wrap her brother's present and $\frac{3}{4}$ metres to wrap her sister's present. Did she buy enough ribbon?

10. Anne just turned 19 years old. How old was she $4\frac{3}{4}$ years ago?

11. Tabitha ate $\frac{1}{4}$ of a sub and Jordan ate $\frac{2}{3}$ of the sub. How much of the sub was left over?

12. Sadia picked $\frac{2}{5}$ of a basket of berries, Tashi picked $\frac{1}{3}$ of a basket, Shafma picked $\frac{4}{7}$ of a basket, and Marzuk picked $\frac{4}{5}$ of a basket. Did all their berries fit into 2 baskets?

13. Soil at a beach consists of sand, clay and salt. If a sample taken is $\frac{3}{4}$ sand and $\frac{1}{6}$ salt, how much of the sample is clay?

14. Gerome has an hour and a half before he has to leave to play basketball. If it takes him $\frac{7}{12}$ of an hour to do his homework and $\frac{3}{5}$ of an hour to find his uniform, how much time will he have left?